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Analysis of Coronavirus Patients Flow in Hospitals: An Application of Queuing Theory

Walaa A. Abdellatief[a], Amira Abdelatey[b].

[a]*Information Technology department, Faculty of Computer and Information, Menoufia University, Egypt, Walaaali285@gmail.com*
[b]*Information System department, Faculty of Computer and Information,Menoufia University , Egypt, amira.mohamed@ci.menofia.edu.eg*

Abstract

During the Coronavirus disease 2019 crisis, many hospitals suffered from a severe shortage of resources. This was due to the exponential growth in the number of recorded cases. In many world countries, COVID-19 patients had to wait for a free bed which puts their lives at risk. Therefore, hospitals are forced to raise the limits of their permissible resources to be able to afford all cases. In this paper, the queuing theory model was used as an operations management technique to calculate the number of needed beds that must be available to cover all recorded cases. Based on the available information, Italy was chosen as a case study - which was one of the hotspot countries in the beginning of the crisis after China. An analysis of the behavior of the waiting lines in hospitals is introduced from the beginning of the crisis and for more than one year later. According to queuing theory formulas, the number of the required hospital beds and ICU beds are determined. Moreover, the probability of waiting and the utilization of the system are also calculated. By testing the system, we concluded that 20000 hospital beds were required to accommodate about 99% of hospital patients recorded in the highest period of infection. In addition, 2500 ICU beds were needed to serve 100% of ICU patients recorded in the same period. The expected waiting delay that patients suffered because of the waiting queues is also measured.

Keywords: COVID-19; Pandemic; Queuing Theory; Beds; Waiting time.

1. Introduction

Coronavirus disease (COVID-19) is a new infectious disease spread in December 2019 from Wuhan, China to the whole world [1]. SARS-CoV-2, the virus that causes COVID-19 infection can be easily transmitted from one person to another. The major difference between COVID-19 and related viruses is its ability to rapidly spread by human contact, or by direct contact with tainted surfaces [2]. The vast majority of people infected by COVID-19 suffer from mild symptoms and can be treated at home. However, some people with a weak immune system or with chronic diseases suffer from severe symptoms and require hospitalization. Most of the hospitalized cases only need normal care beds, while about 30% of them need intensive care beds [3].

With the rapid spread of the disease and consequently the increase in the number of infections, there was a need for models to analyse, develop, and decide the best way to handle hospitals resources to accommodate all expected cases [4]. These models can help decision-makers in healthcare systems to figure out the size of the problem and enable them to handle it.

During pandemic times, the waiting time is the key element for measuring the quality of service in hospitals. Thus, minimizing the waiting time is a major factor for increasing the efficiency of these systems [5]. Therefore, over the years, hospitals management invested great effort to propose solutions to enable them to deal with the high spread of epidemic diseases. Queuing theory was one of the methods that were used in hospitals to provide recommendations for the minimum enough number of resources needed in such situations. This is important because it is not realistic and economically impossible to continuously increase the number of resources in all hospitals.

In hospitalization systems, different queues are present some for low-suspected cases and others for high-suspected cases. Queuing model can be used to analyse the number of queued cases according to the arrival rate of patients and the length of their stay in the hospital. In this paper, the main focus was on the flow of COVID-19 patients in healthcare systems and the occupancy rate during the epidemic using queuing theory model.

The organization of the rest of the paper is done as follows: In section 0, a related work about the queuing theory model and healthcare systems are discussed. Added to that, the previous work that relates to COVID-19 and queuing theory are reviewed. Section 3 previews the structural hospitalization model for COVID-19 cases. The problem description is discussed in section 4. In section 5, experimental results, analysis of the addressed dataset, and discussion for the achieved results are shown. The paper ends with a conclusion in section 6. Through this paper, four contributions are achieved which are: (1) the use of the famous queuing theory model to analyze and give suggestions to one of the current trendy problems caused by COVID-19, (2) calculation of the number of hospital beds and ICU beds in Italy that are required to be available to serve all recorded cases in the period from 24 February 2020 to 23 May 2021, (3) calculation of system utilization that reflects the efficiency of the system and the acceptance rate when beds are not enough, and finally (4) calculation of the average waiting time patients would experience if the beds were not enough and there were waiting for queues.

2. Related Work

COVID-19 has caused ongoing challenges for the healthcare systems all over the world and that's since the end of 2019. The administration of hospitals was felled under immense pressure during the pandemic. Therefore, they adapted their healthcare systems to be able to monitor their performance measures and their quality of service in the crisis. One of the main goals of the hospitals at this time is the guarantee of admission for all patients. Thus, to overcome this crisis and minimize its negative impact, it is supposed to utilize all the available resources. Studies have been done to help hospital managers to achieve this goal by providing predictions for the number of resources needed. In this section, a background is previewed for the queuing theory concepts, the healthcare systems structure and needs, and COVID-19 in healthcare systems with the applied queuing model.

2.1 Queuing Theory

In the early of 1900s, Agner Krarup Erlang, a telephone engineer, began a study of the waiting lines that occurred in the telephone networks where operators have to answer the incoming telephone calls. These waiting lines are known as queues and the analysis of these queues is known as queuing theory [6]. Since then, the queuing theory has been grown in many sophisticated applications with waiting lines such as, supermarkets, petrol stations, computer systems, and the most important of them, which is the subject of this study, are hospitals. This theory concerns the study of constructing a model that is used for predicting the lengths of queues and the time of waiting in a system. Researchers used this model to help managers to make better decisions about the operation of the waiting lines in their systems. Therefore, queuing theory became a field of applied probability and was taken into account when decision-makers have to decide the resources needed to provide the service [7]. The waiting line model consists of mathematical formulas that are used to determine the performance measures of queues. Six performance measurements are used to evaluate the behaviour of waiting queues [8] which are: (1) the probability that there are no patients in the system, (2) the average number of patients in the waiting queue, (3) the average number of patients in the whole system which are the number patients in the queue plus the number of patients being served, (4) the average waiting time for the patient in the queue, (5) the average time a patient spends in the system which is the waiting time plus the service time, and finally (6) the average server utilization. With this information, decision makers are better able to make decisions that balance desirable service levels against the cost of providing the service.

The queuing model is usually described by a shorten form by Kendall's notation [9]. This notation describes the queuing system by specifying the arrival process, the service time distribution, and the number of servers. These systems can have single or multiple servers. Single server examples include small retail stores with a single checkout counter. Multiple server systems have parallel service providers or channels offering the same service. In these systems, any of the servers can offer the service needed by the customers waiting in the customer queue and all servers and all customers are alike [10]. Thus, the process of patient acceptance in hospitals and the reservation of a bed can be represented by Kendall's notation as an M/M/s queue model. This model contains a waiting queue of patients who are waiting for a service from a set of s servers. In hospitals, beds are the servers. Patients who need beds arrive with a Markovian Poisson arrival process and are served with Markovian exponential service time. With high arrival rates, queues of waiting patients appear as shown in Fig 1.

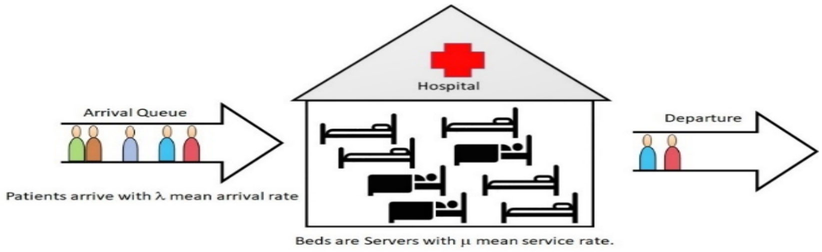

Fig 1. Queuing lines of patients in hospitals

For M/M/s multi-queue, arrival occurs at arrival rate λ according to Poisson distribution. Service rate pattern has an exponential distribution with parameter μ. Population size is infinite and there is no limit to the number of customers. First-Come-First-Served (FCFS) is the queue discipline in this model. For a multi-server system, If the demand for a service is high enough, it may be necessary to add more queues. If the customer waiting time is too high, more servers are needed to improve the performance of the model. On the other hand, if the arrival rate of the model is not high enough, it is not convenient to add more servers as server's utilization will be lower. In the last case, it is recommended to decrease the number of servers on the system.

2.2 *Healthcare systems*

Research papers showed that the queuing theory model can be effectively used in healthcare systems. Examples of some literature reviews in this field are present in [11] and [12]. Samuel and Jaffrey in [11] listed the applications of queuing theory in the field of healthcare. Authors discussed the benefits of using queuing theory in different stages of these systems such as waiting time, utilization analysis, system design, and appointment systems. In [12], different applications of queuing models in the field of healthcare systems management are provided. The review showed that queuing models in healthcare systems can be applied in ambulatory care, pharmacy services, and in multi-hospital departments control. Besides, it can be utilized for resources and patients scheduling. In addition, it can be employed in server's utilization analysis, queues length, and costs minimization.

However, other studies are specific for defined hospital cases. For example, De Bruin et al. in [13] analysed the congestion in the flow of cardiac patients in the emergency care chain using the queuing model. They studied the number of rejected admissions at the First Cardiac Aid. They proposed a queuing model with parameters estimated from the hospital data to provide proper bed allocation and reduce admission rejection. They used a service discipline of first-come first-served model with allowed re-admission. Results also clarified the relation between the variability on the length of stay and the capacity requirement of the hospital. In [14], a study for the government clinics which are suffering from long waiting times is conducted. It analysed the relation between the service demand and the number of doctors in the Emergency Department. Minimum number of doctors leads to a high level of bottleneck, longer waiting times and therefore highest probability of diseases spread between patients in the Emergency Department. A queuing theory model is applied to provide patients satisfaction and better performance. In [15], a simulation of a multi-server queuing model for patient flow in the outpatient department in a health clinic in Malaysia has been developed. The target was to determine the waiting arrival time and service time of patients. The model proved that the longest waiting time exists at the pre-consultation room and the lowest server utilization is at the registration counter. In [16], the authors used the Markov chain concept to construct a transition matrix that describes the queuing states for the hospital beds allocation service. By solving the transition probability matrix of the constructed Markov chain, the model provides the optimal length of the queue and the optimal waiting time under the steady state of the system. Results showed that using Markov optimization enhanced patients' needs. Displayed results showed the change in the resources utilization and the patients waiting times as a function of the queue length.

2.3 COVID-19 and Queuing Theory

Several studies have been conducted on queuing problems with COVID-19 patients since the beginning of the pandemic. In [17], according to a simulation, a graph that shows the relation between the arrival rate and the achieved throughput of patients for reserving ICU beds is present. By making the simulation available online in [18], authors allowed users to set the arrival rate to the ICU, the Length of stay (LOS) distribution specified with median and interquartile range (IQR) or mean and standard deviation (SD), and the number of ICU beds allocated to COVID-19 and non-COVID-19 patients and the tool will output the number of admitted COVID patients per day that get an ICU bed. The tool can deal with arrival rate ranging from 0 to 10 patients per day although the real rate is greater than this range.

Depending on Little's Law [19] which proved that the number of patients in the system equals the mean arrival rate multiplied by the length of time patients waiting in the system, the authors in [20] wants to answer the question: how many ICU beds was required in Australia?. By simulating an exponential growth for the arrival rate for 28-day, authors showed the ICU admission rate for a different number of supposed numbers of ICU beds. They concluded that the number of required ICU beds will be about 50% of the new confirmed cases during the exponential growth phase of a period of the study which is relatively small to judge.

The shortage that is caused by COVID-19 patients to the number of available ventilator machines and the respiratory therapists who operate these machines is discussed in [21]. By using the famous Erlang C queuing model [22], authors calculated the probability of a new patient waiting for a free ventilator, and the desired number of ventilators to be available to achieve a specific acceptable waiting time. The results are shown for three hospitals separately.

In [23] and based on the queuing model, authors studied the shortage in the number of ICU beds in some countries which were Lombardy (Italy), France, Spain, Belgium, New York State (United States), South Korea, and Japan. According to the ratio of arrival to departure, the number of extra needed ICU beds is defined. However, as a result for the lack of information about some countries, calculations for the second and third waves cannot be done.

Through this review, there are some limitations in the previously proposed papers that discuss the queuing problem in COVID-19 pandemic. Some of them deal with small arrival rates that do not reflect the real rates that caused the crisis. Moreover, the results are shown for a small evaluation period which also does not reflect the changes that occurred during this epidemic. Furthermore, additional metrics about the system behaviour have to be shown to give more details about the system efficiency.

3 Structural Coronavirus hospitalization model

During the COVID-19 pandemic, hospitals are heavily loaded with patients. Therefore, the process of hospital management modeling required a series of steps trying to solve the capacity problem. In this section, the main procedures followed in hospitals to deal with the pandemic is outlined. Hospitals all over the world agreed upon some steps to deal with the expected COVID-19 patients. These steps are: (1) online diagnosis, (2) emergency department phase (Triage phase), and (3) hospitalization phase. Online diagnosis is a new step that is added during the pandemic to reduce overcrowding in hospitals. During the pandemic, special emergency department is concerned with patients with COVID-19 symptoms. It is an examination area for expected COVID-19 patients. Hospitalization step is the quarantine area which controls only beds for COVID-19 patients during quarantine. The flow of COVID-19 suspicion patients is clarified in Fig 2.

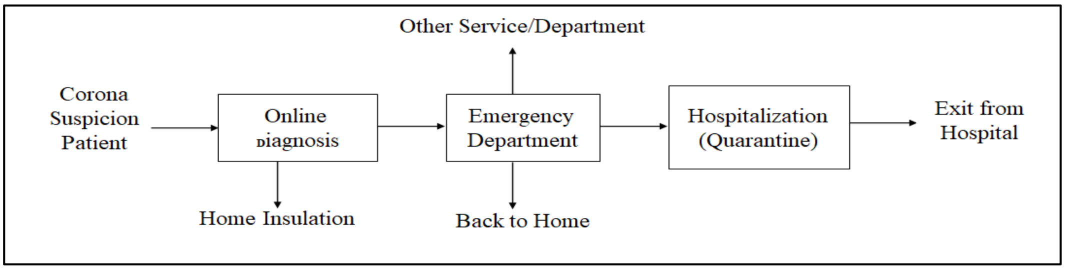

Fig 2. Coronavirus suspicion patients system flow.

The online diagnosis effectively alleviated the emergency phase workload and facilitated the early detection of urgent cases. So, it is an important phase of a pandemic disease. Fig 3 shows the details of the online diagnosis phase to determine if a mild or severe symptom of the patient. People with severe symptoms should have pre-examination at the hospital within the emergency department triage.

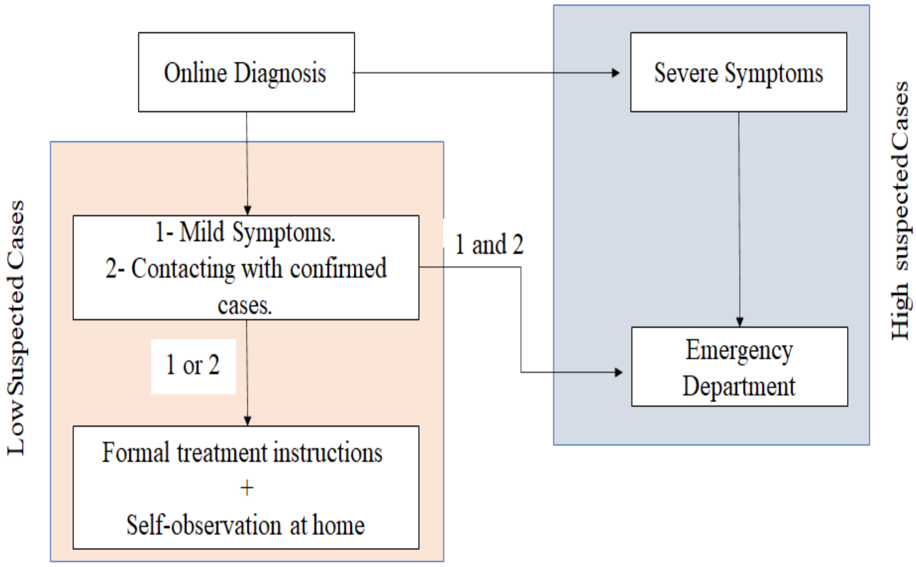

Fig 3. Coronavirus patients online diagnosis.

During the Emergency Department (ED) phase, a pre-examination is conducted on patients. It is a triage step that depends on the degree of severity of the patient's symptoms. Researchers on COVID-19 have defined most mild and severe symptoms [24]. The most common Mild symptoms: are Fever (F), Dry cough (D), Headache (H), and Temperature (T). The most common severe symptoms are Respiratory syndrome (R), Chest pain (c), Loss of speech (S), and Loss of Movement (M). Hospitalization is needed for severe symptoms with PCR test confirmation through normal care room or intensive care room. Fig 4 shows the flow of patients in the ED. This region is separated into low-risk region and high-risk region. This triage reduces the cross-infection by separating the activity of both patient cases. Patients with confirmed infection go to the high-risk region according to their case to be isolated in a normal care room or intensive care room. Patients in the normal care room are assigned a normal hospital bed while patients in intensive care rooms are assigned an ICU bed.

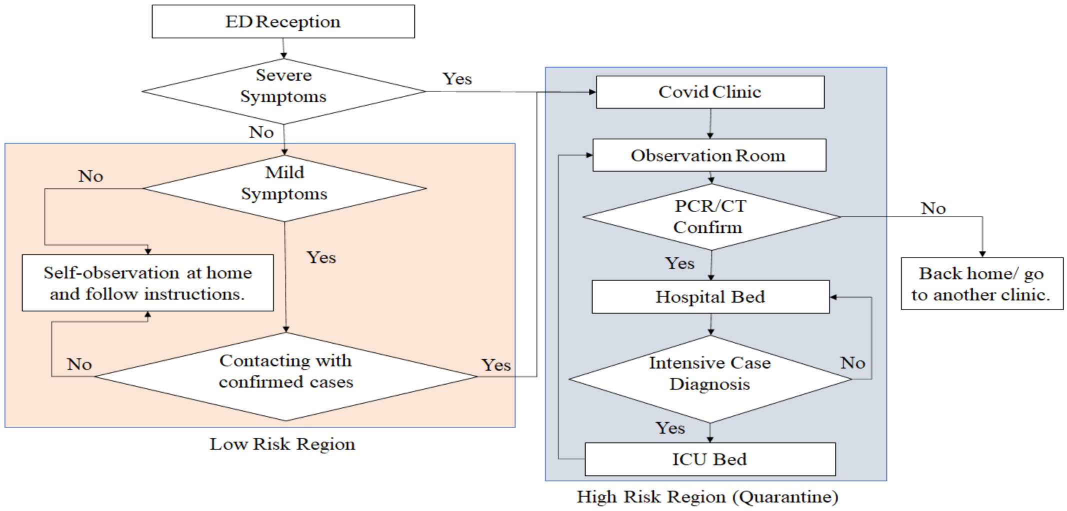

Fig 4. Coronavirus emergency department (ED) patients flow.

Determining the number of needed hospital beds and ICU beds is the main problem during the pandemic. In this paper, the queuing model is used to determine these numbers according to a real recorded patients arrival rate.

4 Problem description

With the reported bed shortages in most of the countries, governments turned to establish filled hospitals to increase the capacity of their beds and accommodate the increasing number of patients. For example, in China, a 1,000-bed capacity filled hospital was built on 24 January 2020 [25]. In Italy on 20 of March 2020, a field hospital was established in three days, to begin with, 8 ICU beds and 20 hospital beds [26]. Then more supply reached to add more 68-bed. In Brazil, in April 2020 a 200-bed field hospital was established [27]. In Egypt, in June 2020 a field hospital with 171 beds, including 11 ICU beds was established [28]. In India, at the time of writing this paper, a second wave of the pandemic started April 2021 with the highest number of daily average recorded cases from the start of the crisis that is 89,800 [29]. The Indian government also mobilized field hospitals to the most affected cities [30].

Therefore, a mathematical model is needed that uses the number of patients and gives suggestions and plans for the required resources (hospitalized beds and ICU beds) to be able to treat all patients. Queuing theory can be used in this situation to model the relation between the number of patients and the available beds in hospitals. In addition, it is used also to study the expected waiting time for a patient to get an available bed. With the increasing arrival rate of patients in some months of the year and a partly long length of stay period, hospitals will suffer from waiting times of patients which is not acceptable. In this paper, the COVID-19 patients' queues with a real dataset have been analyzed. According to the basic queuing theory, The basic parameters that control the queuing model are shown in Table 1.

Table 1. Parameters of the queuing model.

λ	Arrival rate of patients.
LOS	Length of stay.
μ	Service rate.
Beds	Number of servers.

5 Simulation Results and Discussion

Using MATLAB, a simulation for the system is done. Parameters are measured to monitor and evaluate the performance of the system. In this simulation, the used data is released based on the reports issued by the Italian Presidency of the Council of Ministers - Civil Protection Department website "Sito del Dipartimento della Protezione Civile - Emergenza Coronavirus" [31]. This data has also been published by the European Centre for Disease Prevention and Control [32]. The dataset is stored on [33] and viewed on [33]. The dataset records the daily occupancy of COVID-19 patients for the hospital beds and the ICU beds in all provinces of Italy. For example, it was recorded that on 24 February 2020, 127 patients needed hospital beds and 26 patients needed ICU beds. Moreover, the maximum recorded occupancy was 38507 hospital beds on 11 November 2020 and 4068 ICU beds on 4 March 2020.
In this simulation, an evaluation of the data is conducted for the period from 24 February 2020 and up to 23 May 2021. According to [34], a random LOS values with an average of 14 days is used for the hospitalized patients and 8 days for the ICU patients. The target of the simulation is to find a suitable number of beds that is enough to serve all patients who arrived during this period. The effect of the length of the queue is also studied to be able to measure the resulting waiting periods.

To evaluate the system, a testing is conducted for the daily behaviour of the system by measuring the daily number of arrived patients versus the number of accepted and rejected patients. The experiment is done with two scenarios, the first without queuing which means that every patient is accepted if there are enough free beds or rejected otherwise. In the second scenario, a queue is added to the system and some of the patients can wait in the queue for a bed to be free. When the queue is full, any arrived patients are rejected by the system. Also, a measurement for the daily utilization and the daily occupancy versus the daily probability of waiting is directed according to Equations (1), (2), and (3). Both utilization and occupancy are important for evaluating the productivity of the system. The system utilization indicated the percentage of servers that is needed to cover the total arrival rate however some of the patients will be rejected. While the occupancy indicates how many servers from the available are providing service according to the exactly accepted patients. The daily probability of blocking shows all patients that are not served on that day (i.e., queued for another day or rejected from the system). These values are measured for the hospital beds as shown in Fig 5 and for ICU beds as shown in Fig 6.

$$\text{The Daily Utilization} = \frac{\text{Daily Number of Arrived Patients}}{\left(\frac{1}{\text{LOS}}\right) \times \text{Total Number of Available Beds}} \tag{1}$$

$$\text{The Daily Occupancy} = \frac{\text{Daily Number of Reserved Beds}}{\text{Total Number of Available Beds}} \tag{2}$$

$$\text{The Daily Probability of Blocking} = \frac{\text{Daily (Queued + Rejected) Patients}}{\text{Daily Number of Arrived Patients}} \tag{3}$$

In Fig 5 and Fig 6, the behavior of the system on daily basis is presented. Fig 5 concern the case of hospital beds while Fig 6 concern ICU beds.

In Fig 5 in the case of hospital beds, a total of 88591 patients arrived during the period of evaluation. During the whole period, there were three peaks. The first one with the highest number of arrived patients with 2290 patients arrived on day 28, the second was 1446 patients arrived on day 259 and the third was 895 patients arrived on day 386.

Fig 5 is divided into twelve subfigures. The six subfigures named (A1), (A2), (A3), (A4), (A5), and (A6) show the number of daily arrived patients versus the number of daily accepted patients versus the number of daily rejected patients. These values are plotted in the sex subfigures for different numbers of suggested hospital beds and different queue sizes. The other six subfigures named (B1), (B2), (B3), (B4), (B5), and (B6) show the daily utilization versus the daily occupancy versus the daily probability of blocking for each corresponding (A) subfigure.

For more clarification, for example, 2500 beds are used with zero queue size in (A1). 2500 beds are also used in (A2) but with a larger queue size which equals 200. The number of the accepted patients increases from (A1) to (A2) and therefore the daily utilization also increases from (B1) to (B2) using a larger queue size. In (A3), 10000 beds are used, while 15000 beds are used in (A4), and 20000 beds are used in (A5). These three subfigures are plotted with zero queue size. In addition, the number of rejected patients decreases from (A3) to (A4) to (A5) and therefore the probability of blocking also decreases from (B3) to (B4) to (B5) with the increase in the number of available beds. And finally, increasing the number of hospital beds to be 20000 as shown in (A5) and (A6) with zero and 200 queue size respectively. From this, about 99 % of the patients will be accepted.

To summarize, by testing the system with different number of beds, about 10000 hospital beds were enough to accept all patients in the third peak period that was the period with the least total recorded cases. While 20000 hospital bed was required to accommodate about 99% of patients in the first peak period in the most difficult period that has been passed in the country with the highest recorded rate of infections.

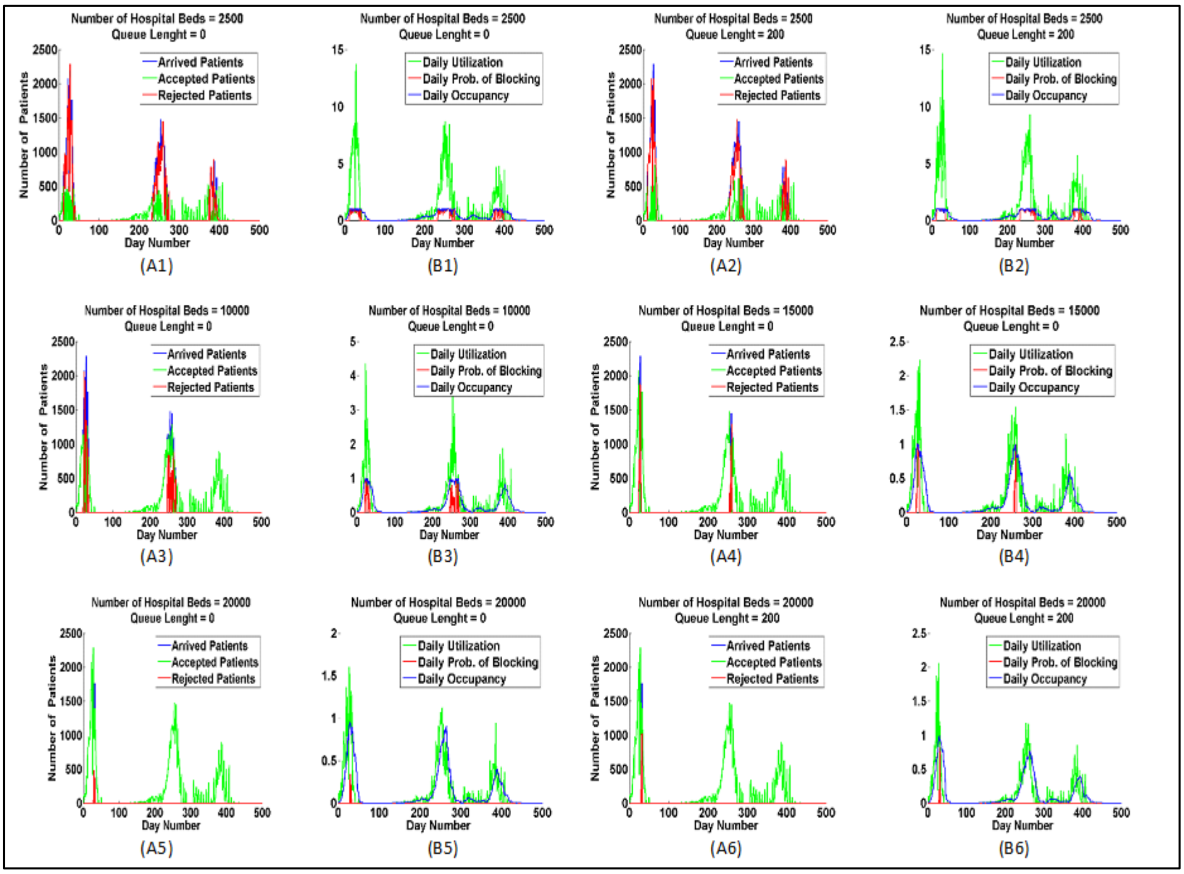

Fig 5. The Daily (Arrived vs Accepted vs Rejected) Number of Patients for Hospital Beds (A1) Beds = 2500 , Queue = 0 (A2) Beds = 2500 , Queue = 200 (A3) Beds = 10000 , Queue = 0 (A4) Beds = 15000 , Queue = 0 (A5) Beds = 20000 , Queue = 0 (A6) Beds = 20000 , Queue = 200 and The Daily (Utilization vs Occupancy vs Prob. of Blocking) (B1) Beds = 2500 , Queue = 0 (B2) Beds = 2500 , Queue = 200 (B3) Beds = 10000 , Queue = 0 (B4) Beds = 15000 , Queue = 0 (B5) Beds = 20000 , Queue = 0 (B6) Beds = 20000 , Queue = 200.

In the case of ICU beds, the daily measured results are shown in Fig 6. It was divided in the same way as Fig 5 with twelve subfigures. In that case, the total number of arrived patients that need ICU beds is 10003. There were also three peak values which are the 241 patients who arrived on day 25, 203 patients on day 254, and 99 patients on day 387.

A testing is started by suggesting the use of 500 ICU beds in (A1) and (A2) and show the effect of changing the queue size from zero to 200 respectively. Besides, the effect of increasing the number of ICU beds is tested from 1000 beds in (A3) to 1500 beds in (A4) to 2500 beds in (A5), all with zero queue size. Increasing the queue size or the number of beds increased the number of accepted patients and so the value of the daily occupancy and the daily utilization. And with the case of the maximum suggested ICU beds which are 2500 and with zero queue size in (A5) and 200 queue size in (A6), 100% of the patients were accepted. Therefore, In case of ICU, 1000 ICU beds was enough to serve all patients during the third peak period, while 2500 ICU beds were required to serve 100% of the patients recorded in the first peak period.

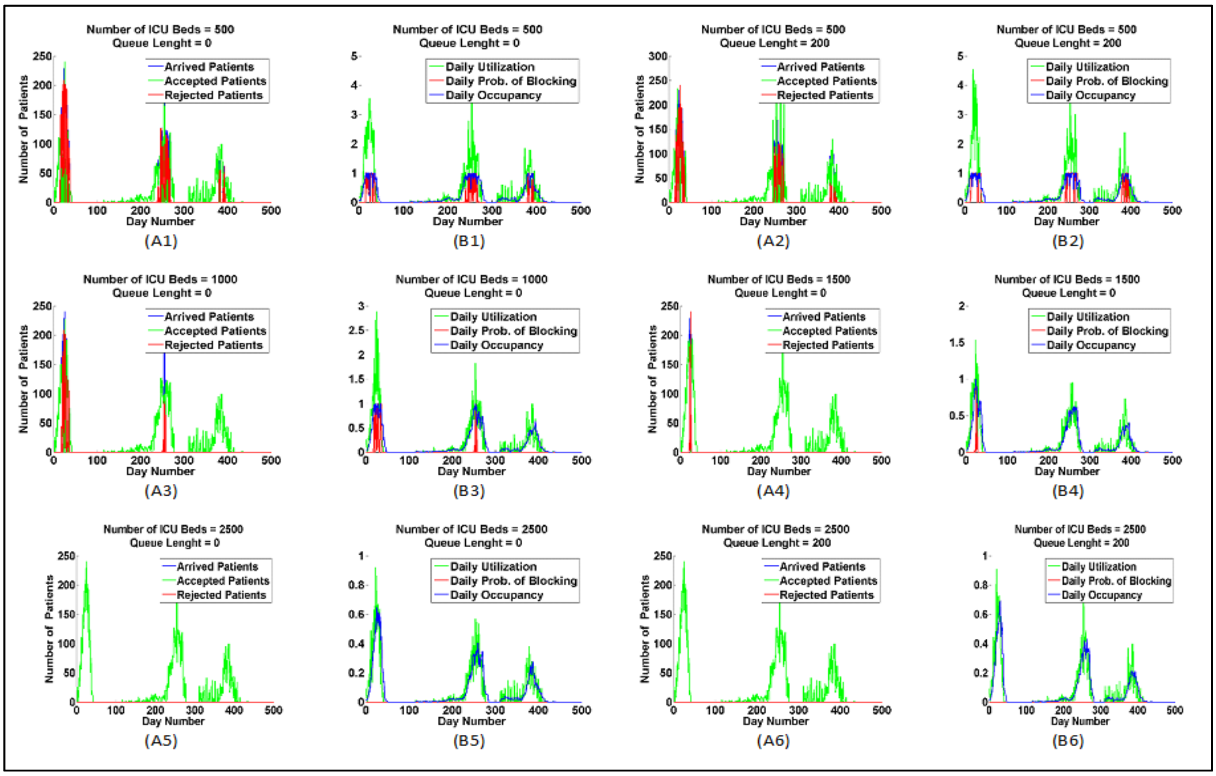

Fig 6. The Daily (Arrived vs Accepted vs Rejected) Number of Patients for ICU Beds (A1) Beds = 500 , Queue = 0 (A2) Beds = 500 , Queue = 200 (A3) Beds = 1000 , Queue = 0 (A4) Beds = 1500 , Queue = 0 (A5) Beds = 2500 , Queue = 0 (A6) Beds = 2500 , Queue = 200 and The Daily (Utilization vs Occupancy vs Prob. of Blocking) (B1) Beds = 500 , Queue = 0 (B2) Beds = 500 , Queue = 200 (B3) Beds = 1000 , Queue = 0 (B4) Beds = 1500 , Queue = 0 (B5) Beds = 2500 , Queue = 0 (B6) Beds = 2500 , Queue = 200.

In the previous two figures, the daily behavior of the system is revealed. Next, the overall behavior of the system in the whole period of study is evaluated. To do that, the total and the average number of accepted, rejected and queued patients for a different number of beds assumed to be available is measured. The system is tested with 2500, 5000, 10000, 15000, and 20000 are used as hospitalized beds and 500, 1000, 1500, 2000, and 2500 are used as ICU beds. Besides, the behavior of the system while using different queue sizes which are 0, 50, 100, 150, and 200 is analyzed. In addition, the probability of waiting time, the average time patients wait for a bed, and the average time patients spend in the hospital which includes waiting plus serving time are investigated. These values are calculated according to equations (4), (5), (6), (7), (8), and (9) and shown in Fig 7 and Fig 8.

$$\text{Average Number of Accepted Patients} = \frac{\text{Total Number of Accepted Patients}}{\text{Total Number of Arrived Patients}} \quad (4)$$

$$\text{Average Number of Rejected Patients} = \frac{\text{Total Number of Rejected Patients}}{\text{Total Number of Arrived Patients}} \quad (5)$$

$$\text{Average Number of Waiting Patients} = \frac{\text{Total Number of Queued Patients}}{\text{Total Number of Arrived Patients}} \quad (6)$$

$$\text{P(Wait)} = \frac{\text{Total Number of Queued Patients}}{\text{Total Number of Arrived Patients}} \quad (7)$$

$$\text{Average Time a Patient Wait} = \frac{\sum \text{Days Patients Spent in the Queue}}{\text{Total Number of Arrived Patients}} \quad (8)$$

$$\text{Average Time a Patient Spends in Hospital} = \frac{\sum \text{(Waiting + Serving) Days in the Hospital}}{\text{Total Number of Arrived Patients}} \quad (9)$$

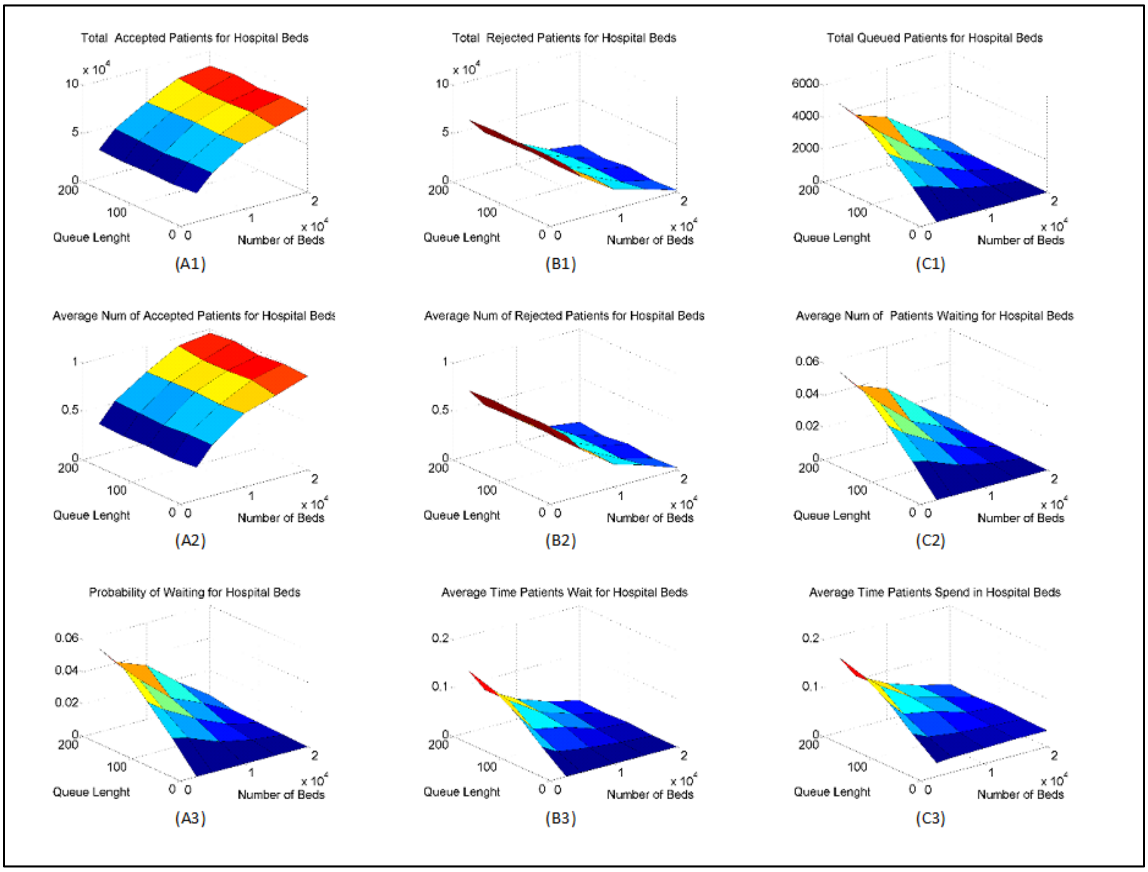

Fig 7. (A1) Total Accepted Patients (B1) Total Rejected Patients (C1) Total Queued Patients (A2) Average Accepted Patients (B2) Average Rejected Patients (C2) Average Queued Patients (A3) Probability of Waiting (B3) Average Time of Waiting (C3) Average Time of Stay in Hospital. For Suggested Hospital Beds 2500; 5000; 10000; 15000; 20000 and Suggested Queue Size 0;50; 100;150; 200.

More specifically, Fig 7 and Fig 8 are drawn to show the metrics measured to monitor the behavior of the system during the whole period of evaluation. The two figures are divided into nine subfigures named (A1), (B1), (C1), (A2), (B2), (C2), (A3), (B3), and (C3). Both figures present the same metrics in the same order except that Fig 7 presents results for hospital beds while Fig 8 presents results for the ICU beds. In (A1), (B1), and (C1) subfigures, the total number of accepted, rejected, and queued patients are presented. In (A2), (B2), and (C2) subfigures, the average number of the accepted, rejected, and queued patients are shown. Finally, (A3) shows the probability of waiting, (B3) shows the average time of waiting, and (C3) shows the average time of staying in the hospital.

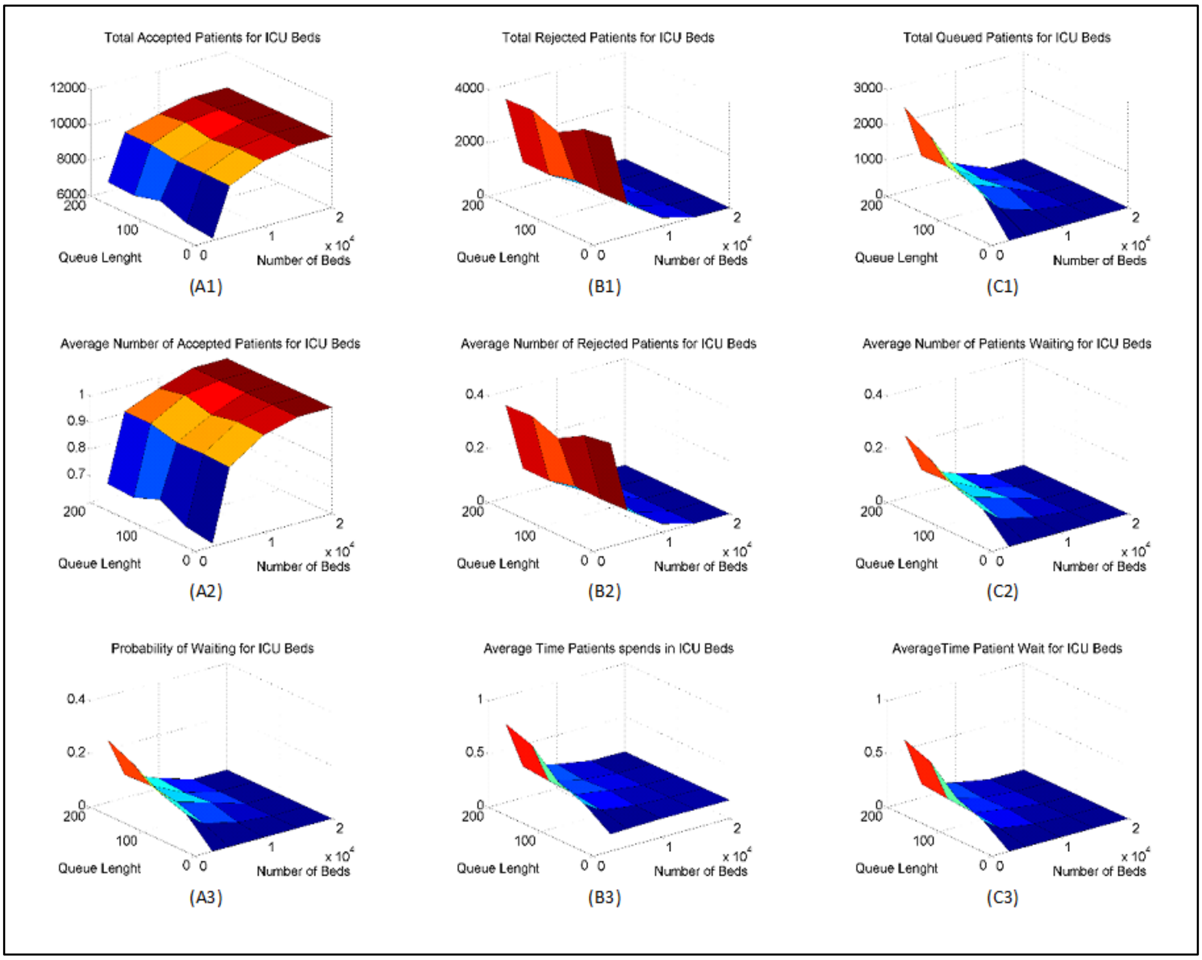

Fig 8. (A1) Total Accepted Patients (B1) Total Rejected Patients (C1) Total Queued Patients (A2) Average Accepted Patients (B2) Average Rejected Patients (C2) Average Queued Patients (A3) Probability of Waiting (B3) Average Time of Waiting (C3) Average Time of Stay in Hospital. For Suggested ICU Beds 500; 1000; 1500; 2000; 2500 and Suggested Queue Size 0;50; 100;150; 200.

To help understand all the plotted values in Fig 7, Table 2 and Table 3 are added to show the percentage of change in the measured metrics with the change in the number of beds or the size of the queue. Table 2 summarizes the percentage of accepted patients from the total recorded patients for each number of suggested hospital beds. It also shows the rate of increase of patients' acceptance respectively. Table 3 shows the effect of the queue length on the average time a patient waits using 15000 and 20000 hospital beds. To illustrate, let's take an example when zero patients are queued, and 2500 hospital beds are used. Table 2 finds that 29556 patients will be accepted which are 33.36% of the total recorded patients. While increasing the number of available hospital beds until reaching 20000 hospital beds, number of 87639 patients will be accepted which are 98.93% of the total recorded patients. Also, when the number of beds is increased from 5000 to 10000, the number of accepted patients increased by 28.4%. Furthermore, to study the effect of queuing, Table 3 is added to show the length of waiting time. The average waiting time increases with a longer queue while it decreases with more beds available with the same queue. For example, if the system allowed queuing for 200 patients on a waiting list, there will be an average waiting time of about 26.01 minutes if 15000 hospital beds were available and 6.50 minutes if 20000 hospital beds were available.

Table 2. Relation between the number of hospital beds and the total accepted patients with zero patients queued.

Num. of Hospital Beds	Total accepted patients	Percentage of accepted patients	The rate of increase in acceptance
2500	29556	33.36%	–––
5000	43894	49.55%	16.2%
10000	69035	77.93%	28.4%
15000	79351	89.57%	11.6%
20000	87639	98.93%	9.4%

Table 3. Relation between the queue length and the average time a patient waits for a hospital bed.

Queue Length	15000 Hospital Beds		20000 Hospital Beds	
	Average waiting time (day)	Average waiting time (minutes)	Average waiting time (day)	Average waiting time (minutes)
50	4.72×10^{-3}	6.79	1.13×10^{-3}	1.63
100	1.02×10^{-2}	14.63	2.26×10^{-3}	3.25
150	1.69×10^{-2}	24.38	4.33×10^{-3}	6.24
200	1.81×10^{-2}	26.01	4.52×10^{-3}	6.50

Similarly, Fig 8 shows the same nine metrics as in Fig 7 but for ICU beds. Table 4 and Table 5 are also added to show the percentage of change in the measured metrics. To clarify further, let's take an example when zero patients are queued, and 500 ICU beds are used. As shown in TABLE 4, number of 6216 patients will be accepted which are 62.14% of the total recorded patients. While increasing the number of available ICU beds until reaching 2500 beds, about 10003 patients will be accepted which are 100% of the total recorded patients. In addition, when the number of beds is increased from 500 to 1000, the number of accepted patients increased by 23.3%. Moreover, to study the effect of queuing, Table 5 and Table 3 is added to show the amount of time patients have to wait in the queue. For example, if 200 patients are queued, patients wait on average 66.65

minutes if 1500 ICU beds were available and zero waiting time if 2500 ICU beds were available. To summarize, about 100% of the patients will be accepted using 2500 ICU beds with zero patients queued and zero waiting time.

Table 4. Relation between the number of ICU beds and the total accepted patients with zero patients queued.

Num. of ICU Beds	Total accepted patients	Percentage of accepted patients	The rate of increase in acceptance
500	6216	62.14%	–––
1000	8542	85.39%	23.3%
1500	9674	96.71%	11.3%
2000	10003	100.00%	3.3%
2500	10003	100.00%	0.0%

Table 5. Relation between the queue length and the average time a patient waits for an ICU bed.

Queue Length	1000 ICU Beds		1500 ICU Beds	
	Average waiting time (day)	Average waiting time (minutes)	Average waiting time (day)	Average waiting time (minutes)
50	5.99×10^{-2}	86.23	1.50×10^{-2}	21.59
100	9.10×10^{-2}	131.00	2.00×10^{-2}	28.79
150	1.0×10^{-1}	155.62	3.90×10^{-2}	56.14
200	1.29×10^{-1}	186.42	4.63×10^{-2}	66.65

6 Conclusion

In conjunction with the emergence of the COVID-19 pandemic, several problems have appeared especially in the medical sector. Hospitals in many parts of the world are overwhelmed with COVID-19 patients. This situation still exists because the pandemic is still evolving with the emergence of new strains. Therefore, according to the data of this crisis, studies must be made to help decision-makers to determine their hospital's needs. This paper addresses the problem of managing the long queues of patients who are waiting for hospital services. The concept of queuing theory is used to give recommendations for planning the number of needed beds in Italy according to the recorded arrival rates of patients. This model can be used by hospital managers to determine the required amount of hospital beds and ICU beds to provide service for all patients. By evaluating the period from 24 February 2020 up to 23 May 2021 and with the assumption of zero queuing, number of 20000 hospital beds are enough to serve 98.93% of the 88591 recorded hospital patients. And 2500 ICU beds are enough to serve 100% of the 10003 recorded ICU patients. In case of the availability of 20000 hospital beds, patients may suffer an average waiting period of about 1.63 minutes if 50 patients are queued and about 6.50 minutes if 200 patients are queued. In the case of ICU beds, if 1500 ICU beds were available, patients wait about 21.59 minutes with 50 queued patients and wait about 66.65 minutes with 200 queued patients. However, if 2000 ICU beds were available, patients will not suffer any waiting time.

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